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ON SOLVING THE PROBLEM OF REFLECTION OF LINEAR WAVES IN A FLUID FROM A POROUS HALF-SPACE SATURATED BY THIS FLUID

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Solutions of the problem of reflection of a stepwise pressure wave in a linearly compressed fluid from a flat boundary of a porous medium of infinite length saturated by the same fluid are obtained in the acoustic approximation. Based on analytical solutions, a numerical analysis is performed to reveal the specific features of the reflected and incident waves, depending on porosity and permeability of the porous half-space.

Key words: *stepwise shock wave, porous half-space, filtration rate, reflected and incident waves.*

Introduction. The issues of wave dynamics in porous saturated media have been of interest since the publications of Frenkel [1] and Biot [2]. The theory of waves for the case where the porous medium contains inhomogeneities in the form of cracks or spherical inclusions was developed in [3–6]. It was first demonstrated in [7] that the specific features of propagation of longitudinal waves in some media saturated by the fluid are largely determined by interphase heat transfer. Interaction of shock waves propagating in a gas with a saturated porous partition of finite thickness was considered in [8, 9]. Some studies of the wave dynamics of saturated porous media, as applied to problems of geoacoustics and mechanisms of intensification of oil production, can also be noted [10–15]. Most theoretical works deal with a dispersion analysis or search for numerical solutions of the initial system of equations of saturated media. Analytical solutions that describe interaction of a stepwise pressure wave with a porous wall are constructed in the present paper for a critical situation, namely, for the case with an incompressible skeleton of the porous medium.

Governing Equations and Their Solutions. We consider the process of reflection of a plane one-dimensional pressure wave in a linearly compressed fluid from a flat boundary with a porous and permeable medium saturated by the same fluid. The skeleton of the porous medium is assumed to be incompressible. The coordinate axis is directed perpendicular to the wave front, and the origin ($x = 0$) of coordinates is located in the plane of the boundary. The equations of motion in the domains of the pure fluid ($x < 0$) and in the porous medium ($x > 0$) can be written in the form [16]

$$\frac{1}{C^2}\frac{\partial p}{\partial t} + \rho_0\frac{\partial u}{\partial x} = 0, \qquad \rho_0\frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0 \qquad (x < 0), \tag{1}$$

$$\frac{m}{C^2}\frac{\partial p}{\partial t} + \rho_0\frac{\partial u}{\partial x} = 0, \quad \rho_0\frac{\partial u}{\partial t} + m\frac{\partial p}{\partial x} = -\frac{m\mu}{k}u \qquad (x > 0).$$

Here u is the velocity (or filtration rate in the domain of the porous medium), p are the pressure perturbations, ρ_0 is the gas density, m and k are the porosity and permeability of the porous medium, C is the velocity of sound

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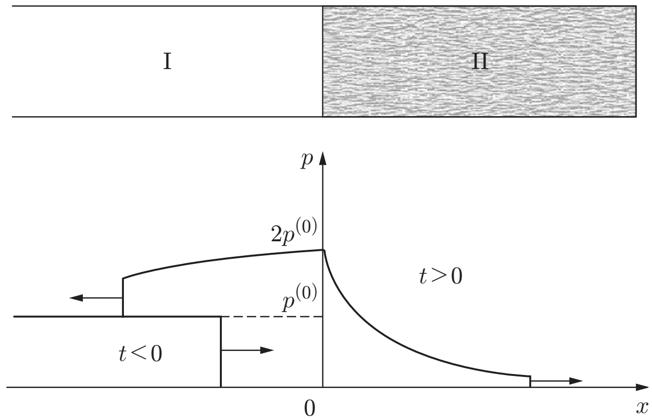

Fig. 1. Wave pattern in the case of shock-wave reflection from a permeable medium: domains I and II are the pure fluid and the porous medium, respectively.

in the fluid, and μ is the fluid viscosity. We assume that the fluid in the initial state in the porous medium is motionless and the pressure is uniform:

$$u = 0, \qquad p = 0 \qquad (x > 0, \quad t \leq 0).$$

Let a pressure wave be incident at the time $t = 0$ onto the surface of the porous medium from the side of the pure fluid; $p^{(0)}(t)$ is the law of variation of the pressure perturbation on the wall surface. Then, the velocity of fluid motion under the action of this wave obeys the relation within the framework of the linear theory

$$u^{(0)}(t) = p^{(0)}(t)/(\rho_0 C). \tag{2}$$

When the wave front reaches (at the time $t = 0$) the surface $x = 0$, a reflected wave is formed in the pure fluid zone:

$$p^{(r)} = p^{(r)}(t + x/C), \qquad u^{(r)} = -p^{(r)}(t + x/C)/(\rho_0 C). \tag{3}$$

Therefore, the total pressure perturbations and velocities in this domain ($x < 0$) for $t > 0$ can be written as

$$p = p^{(0)}(t - x/C) + p^{(r)}(t + x/C), \qquad u = [p^{(0)}(t - x/C) + p^{(r)}(t + x/C)]/(\rho_0 C). \tag{4}$$

The wave pattern formed in the case of shock-wave reflection from a permeable partition of infinite thickness is schematically shown in Fig. 1.

It follows from Eqs. (1) that the perturbations of pressure $p^{(g)}$ and velocity $u^{(g)}$ in the domain of the porous medium are related as

$$u^{(g)}(x,t) = -\frac{m}{\rho_0} \int\limits_0^t \frac{\partial p^{(g)}(x,t')}{\partial x} \, \mathrm{e}^{(t'-t)/t_v} \, dt', \qquad t_v = \frac{\rho_0 k}{m\mu}. \tag{5}$$

In the course of interaction of the pressure wave with the perturbation boundary, the pressures and velocities of the fluid on the side of the pure fluid (left boundary) and on the side of the fluid in the porous medium (right boundary) have to be identical. Then the boundary $x = 0$ for $t \geq 0$ obeys the relations

$$P(t) = p^{(0)}(t) + p^{(r)}(t) = p^{(g)}(0,t), \qquad U(t) = u^{(0)}(t) + u^{(r)}(t) = u^{(g)}(0,t),$$

where $P(t)$ and $U(t)$ are the total perturbations of pressure and velocity at the boundary $x = 0$. With allowance for Eqs. (2)–(5), we obtain

$$2p^{(0)} - P(t) = \rho_0 C U(t), \qquad U(t) = -\frac{m}{\rho_0} \int\limits_0^t \frac{\partial p^{(g)}(0,t')}{\partial x} \, \mathrm{e}^{(t'-t)/t_v} \, dt'. \tag{6}$$

Equations (1) with $p^{(g)}$ yield the equation

$$\frac{\partial^2 p^{(g)}}{\partial t^2} + \frac{1}{t_v} \frac{\partial p^{(g)}}{\partial t} = C^2 \frac{\partial^2 p^{(g)}}{\partial x^2},$$

whose solution satisfying the initial and boundary conditions

$$p^{(g)} = 0, \quad \frac{\partial p^{(g)}}{\partial t} = 0 \quad (x > 0,\ t = 0), \qquad p^{(g)} = P(t) \quad (x = 0,\ t > 0)$$

can be obtained by the Laplace transform method in the form [17]

$$p^{(g)}(t,x) = \frac{1}{2\pi i} \int\limits_{\sigma-i\infty}^{\sigma+i\infty} \Big(\int\limits_0^\infty P(t')\, \mathrm{e}^{-\lambda t'}\, dt' \Big)\, \mathrm{e}^{\lambda t - k(\lambda) x}\, d\lambda, \qquad k(\lambda) = \frac{\sqrt{\lambda t_v (1 + \lambda t_v)}}{C t_v}. \tag{7}$$

From here we obtain

$$\frac{\partial p^{(g)}(0,t)}{\partial x} = -\frac{1}{2\pi i} \int\limits_0^\infty \Big(\int\limits_{\sigma-i\infty}^{\sigma+i\infty} k(\lambda)\, \mathrm{e}^{\lambda(t-t')}\, d\lambda \Big) P(t')\, dt'. \tag{8}$$

Substituting Eq. (8) into the right side of the expression for $U(t)$ from Eq. (6), we obtain

$$U(t) = \frac{m}{2\pi i \rho_0} \int\limits_0^t \Big[\int\limits_0^\infty \Big(\int\limits_{\sigma-i\infty}^{\sigma+i\infty} k(\lambda)\, \mathrm{e}^{\lambda(t''-t')}\, d\lambda \Big) P(t')\, dt' \Big]\, \mathrm{e}^{(t''-t)/t_v}\, dt''. \tag{9}$$

We change the order of integration in (9), namely, we write

$$U(t) = \frac{m\, \mathrm{e}^{-t/t_v}}{2\pi i \rho_0} \int\limits_0^\infty \Big[\int\limits_{\sigma-i\infty}^{\sigma+i\infty} \Big(\int\limits_0^t \mathrm{e}^{t''(\lambda + 1/t_v)}\, dt'' \Big) k(\lambda)\, \mathrm{e}^{-\lambda t'}\, d\lambda \Big] P(t')\, dt'. \tag{10}$$

Then, expression (10) can be presented as

$$U(t) = \frac{m}{\rho_0 C} \int\limits_0^\infty \Big(\frac{1}{2\pi i} \int\limits_{\sigma-i\infty}^{\sigma+i\infty} \varphi(\lambda)\, \mathrm{e}^{\lambda(t-t')}\, d\lambda \Big) P(t')\, dt', \qquad \varphi(\lambda) = \sqrt{\frac{\lambda t_v}{1 + \lambda t_v}}. \tag{11}$$

Using the obvious equality

$$\frac{1}{2\pi i} \int\limits_{\sigma-i\infty}^{\sigma+i\infty} \varphi(\lambda)\, \mathrm{e}^{\lambda(t-t')}\, d\lambda = -\frac{\partial}{\partial t'} \frac{1}{2\pi i} \int\limits_{\sigma-i\infty}^{\sigma+i\infty} \varphi(\lambda)\, \mathrm{e}^{\lambda(t-t')}\, \frac{d\lambda}{\lambda}, \tag{12}$$

we calculate the integral under the sign of the derivative with respect to t' in the right side of Eq. (12) by the known formula [18]

$$\frac{1}{2\pi i} \int\limits_{\sigma-i\infty}^{\sigma+i\infty} \sqrt{\frac{\lambda + 2\beta}{\lambda + 2\alpha}}\, \mathrm{e}^{\lambda t}\, \frac{d\lambda}{\lambda} = \Big(\mathrm{e}^{-\rho t}\, I_0(rt) + 2\beta \int\limits_0^t \mathrm{e}^{-\rho\tau}\, I_0(r\tau)\, d\tau \Big) H(t), \tag{13}$$

$$H(t) = \begin{cases} 0, & t < 0, \\ 1, & t > 0, \end{cases} \qquad \rho = \alpha + \beta, \qquad r = \alpha - \beta,$$

where $H(t)$ is the Heaviside function and $I_0(z)$ is the Bessel function of the imaginary argument. In the case considered, $2\alpha = 1/t_v$ and $\beta = 0$; hence, we have

$$\frac{1}{2\pi i} \int\limits_{\sigma-i\infty}^{\sigma+i\infty} \varphi(\lambda)\, \mathrm{e}^{\lambda(t-t')}\, \frac{d\lambda}{\lambda} = \mathrm{e}^{-(t-t')/(2t_v)}\, I_0\Big(\frac{t-t'}{2t_v} \Big) H(t-t'). \tag{14}$$

Substituting Eq. (14) into the right side of Eq. (12) and involving Eq. (11), we obtain

$$U(t) = \frac{m}{\rho_0 C} \int\limits_0^t \frac{\mathrm{e}^{-(t-t')/(2t_v)}}{2t_v} \Big[I_1\Big(\frac{t-t'}{2t_v} \Big) - I_0\Big(\frac{t-t'}{2t_v} \Big) \Big] P(t')\, dt' + \frac{m}{\rho_0 C}\, P(t). \tag{15}$$

Substituting Eq. (15) into the first equation from (6), we obtain the following integral Volterra equation of the second kind of the convolution type for $P(t)$:

$$2p^{(0)}(t) - (1+m)P(t) = m\int\limits_0^t \frac{\mathrm{e}^{-(t-t')/(2t_v)}}{2t_v}\Big[I_1\Big(\frac{t-t'}{2t_v}\Big) - I_0\Big(\frac{t-t'}{2t_v}\Big)\Big]P(t')\,dt'. \tag{16}$$

If we perform the substitution of the variable $z = (t-t')/(2t_v)$ in the integral, the integral equation (16) acquires the form

$$2p^{(0)}(t) - (1+m)P(t) = m\int\limits_0^{t/(2t_v)} [I_1(z) - I_0(z)]\,\mathrm{e}^{-z}\,P(t-2zt_v)\,dz. \tag{17}$$

For the function $I_\nu(z)$, we have the expansion

$$I_\nu(z) = \sum_{k=0}^{\infty}\frac{(z/2)^{2k+\nu}}{\Gamma(k+\nu+1)k!}. \tag{18}$$

As $z \to 0$, we obtain

$$I_0(0) = 1, \qquad I_1(z) \approx z/2.$$

From the integral equation (17) with the limiting transition as $t \to 0$, we have

$$P(0) = 2p^{(0)}(0)/(1+m).$$

Let the amplitude of the incident wave have a finite limit $p^{(0)}(\infty)$ as $t \to \infty$. Passing to the limit as $t \to \infty$ in the integral equation (17), we obtain

$$2p^{(0)}(\infty) - (1+m)P(\infty) = m\int\limits_0^{\infty}[I_1(z) - I_0(z)]\,\mathrm{e}^{-z}\,P(\infty)\,dz. \tag{19}$$

Taking into account that

$$[I_1(z) - I_0(z)]\,\mathrm{e}^{-z} = [\mathrm{e}^{-z}\,I_0(z)]', \qquad I_0(z) = \frac{\mathrm{e}^z}{\sqrt{2\pi z}}\,[1 + O(1/z)], \qquad z \to \infty,$$

we obtain the following relation from Eq. (19):

$$P(\infty) = 2p^{(0)}(\infty).$$

Equation (16) in the general form is a linear integral Volterra equation of the second kind of the convolution type with the kernel

$$K(t-t') = \mathrm{e}^{-(t-t')/(2t_v)}\,\Big[I_1\Big(\frac{t-t'}{2t_v}\Big) - I_0\Big(\frac{t-t'}{2t_v}\Big)\Big].$$

Applying the Laplace transform to Eq. (16), we obtain

$$(1+m)\tilde{P}(\lambda) = 2\tilde{p}^{(0)} - \frac{m}{2t_v}\,\tilde{K}(\lambda)\tilde{P}(\lambda), \qquad \tilde{P}(\lambda) = \int\limits_0^{\infty} P(t)\,\mathrm{e}^{-\lambda t}\,d\lambda,$$

$$\tilde{p}^{(0)} = \int\limits_0^{\infty} p^{(0)}(t)\,\mathrm{e}^{-\lambda t}\,d\lambda, \qquad \tilde{K}(\lambda) = 2t_v\int\limits_0^{\infty}\Big[I_0\Big(\frac{t}{2t_v}\Big)\,\mathrm{e}^{-t/(2t_v)}\Big]'\,\mathrm{e}^{-\lambda t}\,dt. \tag{20}$$

Integrating by parts, we present the kernel of the integral equation as

$$\tilde{K}(\lambda) = 2t_v\Big[\lambda\int\limits_0^{\infty} I_0\Big(\frac{t}{2t_v}\Big)\,\mathrm{e}^{-(1/(2t_v)+\lambda)t}\,dt - 1\Big].$$

Using the formula [18]

$$\int_0^\infty I_n(\alpha z)\,\mathrm{e}^{-\beta z}\,dz = \frac{(\beta - \sqrt{\beta^2 - \alpha^2}\,)^n}{\alpha^n \sqrt{\beta^2 - \alpha^2}},$$

we can write the final expression for $\tilde{K}(\lambda)$:

$$\tilde{K}(\lambda) = 2t_v(\varphi(\lambda) - 1). \tag{21}$$

Using Eq. (21), we obtain the expression for $\tilde{P}(\lambda)$ on the basis of Eq. (20):

$$\tilde{P}(\lambda) = \frac{2\tilde{p}^{(0)}}{1 + m\varphi(\lambda)}. \tag{22}$$

Let the incident wave have a stepwise form: $p^{(0)}(t) = p^{(0)} = \text{const}$. Then, instead of (22), we can write

$$\tilde{P}(\lambda) = \frac{2p^{(0)}}{[1 + m\varphi(\lambda)]\lambda}.$$

Passing to the inverse Laplace transform, we find the solution for Eq. (16):

$$P(t) = \frac{p^{(0)}}{\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} \frac{\mathrm{e}^{\lambda t}\,d\lambda}{[1 + m\varphi(\lambda)]\lambda}. \tag{23}$$

To calculate this integral, we present the integrand as

$$[1 + m\varphi(\lambda)]^{-1} = A(\lambda) - B(\lambda),$$

$$A(\lambda) = 1 + \frac{m^2\lambda t_v}{1 + (1 - m^2)\lambda t_v}, \qquad B(\lambda) = m\frac{\sqrt{\lambda t_v(1 + \lambda t_v)}}{1 + (1 - m^2)\lambda t_v}.$$

Then the expression for $P(t)$ can be written as

$$P(t) = 2p^{(0)}(J_1 - J_2), \qquad J_1 = \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} \frac{A(\lambda)\,\mathrm{e}^{\lambda t}}{\lambda}\,d\lambda, \qquad J_2 = \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} \frac{B(\lambda)\,\mathrm{e}^{\lambda t}}{\lambda}\,d\lambda. \tag{24}$$

For the first integral, we readily obtain the expression

$$J_1 = 1 + \frac{m^2\,\mathrm{e}^{-\gamma t/t_v}}{1 - m^2}, \tag{25}$$

where $\gamma = 1/(1 - m^2)$. The second integral can be presented as

$$J_2 = \frac{m^2}{1 - m^2}\,\frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} \frac{\sqrt{\lambda t_v(1 + \lambda t_v)}}{(\lambda t_v + \gamma)\lambda t_v}\,\mathrm{e}^{\lambda t}\,d\lambda t_v.$$

Using Eq. (13), we find

$$J_2 = \frac{m}{1 - m^2}\Big[\mathrm{e}^{-t/(2t_v)}\,I_0\Big(\frac{t}{2t_v}\Big) - \frac{m^2}{1 - m^2}\,\mathrm{e}^{-\gamma t/t_v}\int_0^{t/t_v} \mathrm{e}^{(\gamma - 1/2)\tau}\,I_0\Big(\frac{\tau}{2}\Big)\,d\tau\Big]. \tag{26}$$

Substituting the expressions for J_1 and J_2 from Eqs. (25) and (26) into Eq. (24), we obtain the following presentation of solution (23):

$$P(t) = 2p^{(0)}\Big[1 + \frac{m^2\,\mathrm{e}^{-\gamma t/t_v}}{1 - m^2} - \frac{m\,\mathrm{e}^{-t/(2t_v)}}{1 - m^2}\,I_0\Big(\frac{t}{2t_v}\Big) + \frac{m^3\,\mathrm{e}^{-\gamma t/t_v}}{(1 - m^2)^2}\int_0^{t/t_v} \mathrm{e}^{(\gamma - 1/2)\tau}\,I_0\Big(\frac{\tau}{2}\Big)\,d\tau\Big]. \tag{27}$$

Based on solution (27), we can find an asymptotic solution $P(t)$ as $t \to 0$ ($t \ll t_v$). For this purpose, we have to find the first two terms of the Maclaurin series for this function:

$$P(t) = P(0) + P'(0)t + o(t), \qquad P(0) = 2p^{(0)}/(1 + m). \tag{28}$$

From Eq. (27), we can easily obtain

$$P'(0) = \frac{p^{(0)}m}{(1+m)^2 t_v}. \tag{29}$$

Substituting Eq. (29) into Eq. (28), we obtain the asymptotic solution $P(t)$ for $t \to 0$ ($t \ll t_v$):

$$P(t) = \frac{2p^{(0)}}{1+m}\Big(1 + \frac{mt}{(1+m)2t_v}\Big).$$

Using formula (18) for $I_0(z)$, we can readily obtain an asymptotic solution for $P(t)$ as $t \to \infty$ ($t \gg t_v$), based on solution (27):

$$P(t) = 2p^{(0)}\Big(1 - \frac{m}{\sqrt{\pi}}\sqrt{\frac{t_v}{t}}\Big). \tag{30}$$

In the numerical implementation of solution (27), we use the following integral presentation of the Bessel function [18]:

$$I_0(z) = \frac{1}{\pi}\int\limits_{-1}^{1} \frac{\mathrm{e}^{zy}\, dy}{\sqrt{1-y^2}}.$$

Then the integral complex in the last term in Eq. (27) can be written as

$$\int\limits_0^{t/t_v} \mathrm{e}^{(\gamma-1/2)\tau}\, I_0\Big(\frac{\tau}{2}\Big)\, d\tau = \frac{2}{\pi}\int\limits_{-1}^{1} \frac{\mathrm{e}^{(2\gamma-1+y)t/(2t_v)}\, dy}{\sqrt{1-y^2}\,(2\gamma-1+y)} - \frac{2}{\pi}\int\limits_{-1}^{1} \frac{dy}{\sqrt{1-y^2}\,(2\gamma-1+y)}. \tag{31}$$

The last integral in Eq. (31) with the use of the known tabulated integral [19]

$$\int\limits_0^{\pi} \frac{\cos(nx)\, dx}{1 + a\cos x} = \frac{\pi}{\sqrt{1-a^2}}\Big(\frac{\sqrt{1-a^2}-1}{a}\Big)^n \qquad (a^2 < 1)$$

can be calculated and has the form

$$\frac{2}{\pi}\int\limits_{-1}^{1} \frac{dy}{\sqrt{1-y^2}\,(2\gamma-1+y)} = \frac{\pi(1-m^2)}{2m}. \tag{32}$$

Substituting Eq. (31) with allowance for Eq. (32) into Eq. (27), we finally obtain a presentation of the solution convenient for numerical analysis:

$$P(t) = 2p^{(0)}\Big(1 - \frac{m}{\pi}\int\limits_{-1}^{1} \sqrt{\frac{1-y}{1+y}}\, \frac{\mathrm{e}^{-(y+1)t/(2t_v)}\, dy}{(m^2-1)y + m^2 + 1}\Big). \tag{33}$$

In these calculations, we used the formula [20]

$$\int\limits_{-1}^{1} \sqrt{\frac{1-y}{1+y}}\, f(y)\, dy = \frac{4\pi}{2n+1}\sum_{k=1}^{n} \sin^2\Big(\frac{\pi k}{2n+1}\Big) f\Big(\cos\frac{2\pi k}{2n+1}\Big) + R_n,$$

$$R_n = \frac{\pi}{2^n(2n)!}\, f^{(2n)}(\xi), \qquad -1 \le \xi \le 1.$$

Figure 2 shows the dependences $p/p^{(0)}(t/t_v)$. In the case of reflection of a stepwise pressure wave, the pressure perturbation at the boundary of the porous half-space ($x = 0$) is seen to increase asymptotically from the value $P(0) = 2p^{(0)}/(1+m)$ to the value $P(\infty) = 2p^{(0)}$, following the law described by Eq. (30). Thus, the amplitude of the leading shock of the reflected wave and the value of "undercompression"

$$\Delta P_0 = P(\infty) - P(0) = 2p^{(0)}m/(1+m)$$

are determined by the partition porosity (value of m) only. The characteristic time needed to reach the asymptotic value $P(t)/p^{(0)} = 2$ for known values of porosity m, viscosity μ, and density ρ_0 of the fluid, as well as fluid compressibility [determined by the velocity of sound in accordance with the formula for t_v in (5)], depends only on permeability of the porous medium.

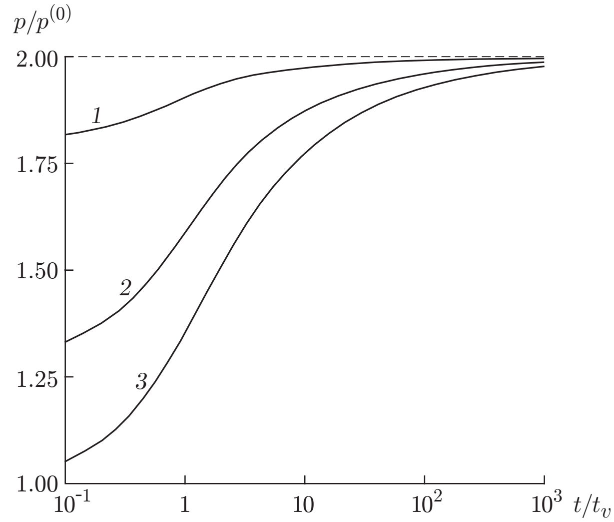

Fig. 2. Pressure in a porous medium with different values of porosity: $m = 0.1$ (1), 0.5 (2), and 0.9 (3); the dashed line is the asymptotic value for curves 1–3.

Let us determine the characteristic relaxation time t_* as a period when the "undercompression" is

$$\Delta P_* = P(\infty) - P(t_*) = \varepsilon \Delta P_0 \qquad (\varepsilon \ll 1).$$

Then, using the asymptotic formula (30) for $P(t)$, we find the expression for the relaxation time:

$$t_* = (1+m)^2 t_v/(\pi\varepsilon^2).$$

In particular, if the fluid consists of air and water with a temperature $T_0 = 300$ K and pressure $p_0 = 0.1$ MPa, the values obtained for a porous medium with a porosity $m = 10^{-1}$ and permeability $k = 10^{-12}$ m^2 (typical of rocks) are $t_v \approx 10^{-6}$ and 10^{-5} sec, respectively.

Let the "undercompression" be reduced by a factor of 10 during the relaxation time t_* ($\varepsilon = 10^{-1}$). Then, we obtain $t_* \approx 3 \cdot 10^{-5}$ and $3 \cdot 10^{-4}$ sec for air and water, respectively. In addition, the characteristic linear scales $x_* = Ct_*$ of relaxation zones in the wave reflected from the porous medium for air ($C = 340$ m/sec) and water ($C = 1500$ m/sec) are $x_* = 0.01$ and 0.50 m, respectively. If less viscous fluids are used (e.g., acetone, benzene, or ethyl alcohol), the characteristic time t_* and the distance x_* are even greater. The estimates show that these results can be used to develop methods of express analysis of porosity and permeability of solid porous materials with the help of pressure waves.

Let us analyze the solution for $p^{(g)}$ and $u^{(g)}$, which describes the wave dynamics in a porous medium. Solution (7) for $p^{(g)}$ corresponding to a stepwise incident wave can be presented as

$$p^{(g)}(x,t) = \frac{p^{(0)}}{\pi i} \int\limits_{\sigma - i\infty}^{\sigma + i\infty} \frac{\mathrm{e}^{\lambda t - k(\lambda)x}\, d\lambda}{[1 + m\varphi(\lambda)]\lambda}. \tag{34}$$

The numerator of the integrand in Eq. (34) can be presented as

$$\mathrm{e}^{\lambda t - k(\lambda)x} = \mathrm{e}^{\lambda(t - x/C) + (\lambda/C - k(\lambda))x}.$$

Using the delay theorem [18], we use Eq. (34) to obtain

$$p^{(g)} = H\Big(t - \frac{x}{C}\Big) \frac{p^{(0)}}{\pi i} \int\limits_{\sigma - i\infty}^{\sigma + i\infty} \frac{\mathrm{e}^{\lambda t' + (\lambda/C - k(\lambda))x}\, d\lambda}{[1 + m\varphi(\lambda)]\lambda}, \qquad t' = t - \frac{x}{C}. \tag{35}$$

We can easily show that the integrand in Eq. (35)

$$\Phi(\lambda) = \mathrm{e}^{(\lambda/C - k(\lambda))x} \Big/ \Big[\Big(1 + m\,\frac{\lambda t_v}{\sqrt{\lambda t_v(1 + \lambda t_v)}}\Big)\lambda\Big]$$

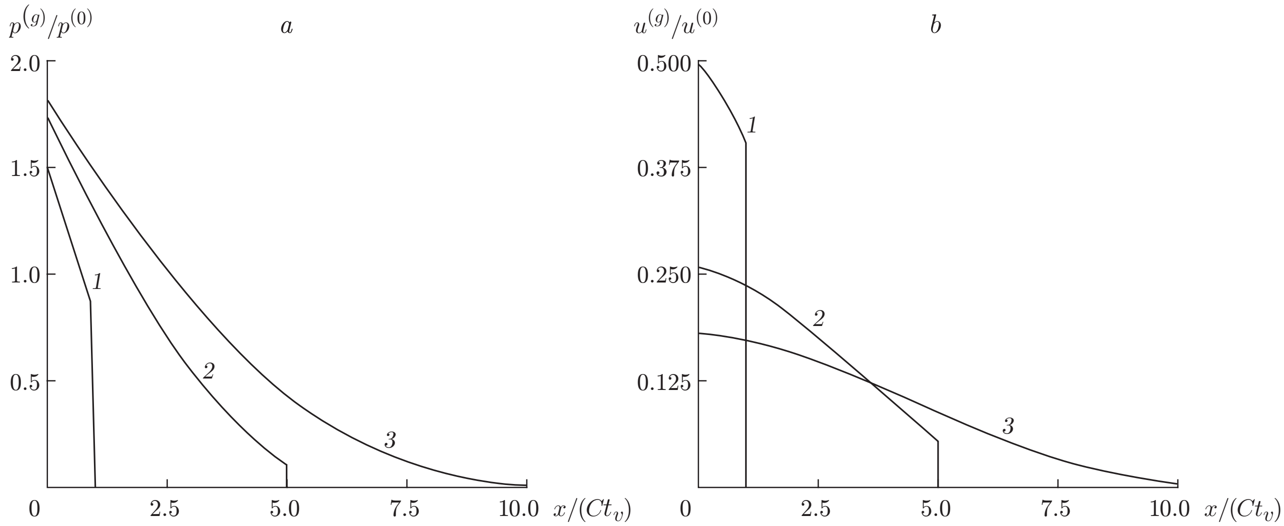

Fig. 3. Evolution of pressure waves (a) and distribution of velocity in a porous medium (b) for $m = 0.5$ and $t/t_v = 1$ (1), 5 (2), and 10 (3).

satisfies the Jordan lemma condition [17]

$$\lim_{\substack{|\lambda|\to\infty \\ \lambda\notin(-\infty,0)}} \Phi(\lambda) = 0.$$

Therefore, applying the method of contour integration to Eq. (35), we obtain

$$\frac{p^{(g)}(x,t)}{p^{(0)}} = 2H\Big(t - \frac{x}{C}\Big)\Big(1 - \frac{m}{\pi}\int\limits_{-1}^{1}\sqrt{\frac{1-y}{1+y}}\,\frac{\cos\,[\sqrt{1-y^2}x/(2Ct_v)]\,\mathrm{e}^{-(1+y)t/(2t_v)}\,dy}{(m^2-1)y+m^2+1}$$
$$-\frac{1}{\pi}\int\limits_{-1}^{1}\frac{1-y}{1+y}\,\frac{\sin[\sqrt{1-y^2}x/(2Ct_v)]\,\mathrm{e}^{-(1+y)t/(2t_v)}\,dy}{(m^2-1)y+m^2+1}\Big). \tag{36}$$

Substituting Eq. (36) into Eq. (5) and applying some simple transformations, we obtain the solution for velocity in a porous medium

$$\frac{u^{(g)}(x,t)}{u^{(0)}} = \frac{2m}{\pi}\,H\Big(t-\frac{x}{C}\Big)$$
$$\times\Big(\pi\,\mathrm{e}^{(t-x/C)/t_v} + \int\limits_{-1}^{1}\sqrt{\frac{1-y}{1+y}}\,\frac{\cos\,[\sqrt{1-y^2}x/(Ct_v)][\mathrm{e}^{-(1+y)t/(2t_v)} - (m+1)\,\mathrm{e}^{-[t-(1-y)x/(2C)]/t_v}]\,dy}{(m^2-1)y+m^2+1}$$
$$-\int\limits_{-1}^{1}\frac{1-y}{1+y}\,\frac{\sin\,[\sqrt{1-y^2}x/(Ct_v)]\,\mathrm{e}^{-[t-(1-y)x/(2C)]/t_v}\,dy}{(m^2-1)y+m^2+1}$$
$$-m\int\limits_{-1}^{1}\frac{\sin\,[\sqrt{1-y^2}x/(Ct_v)](\mathrm{e}^{-(1+y)t/(2t_v)} - \mathrm{e}^{-[t-(1-y)x/(2C)]/t_v})\,dy}{(m^2-1)y+m^2+1}\Big). \tag{37}$$

Figure 3 shows the time evolution of pressure waves and velocity fields in a porous medium, which were calculated by Eqs. (36) and (37) with $m = 0.5$. The quadrature formula (33) was used in the calculations. It follows from Fig. 3 that the leading shock of the pressure wave propagating with a velocity C completely decays during the time $t \approx 10t_v$. For $t \gg 10t_v$, the pressure and velocity fields acquire the form described by the piezoconductivity equation derived within the framework of the Darcy law.

It should be noted that the results obtained by formulas (33), (36), and (37) are in good agreement with the results of the numerical solution of differential equations [8, 9]. In addition, as $t_v \to \infty$, relation (33) coincides with the analytical solution given in [8].

Conclusions. An analytical solution that describes the reflection of a stepwise wave from a porous half-space is obtained in the linear approximation. In contrast to the law of wave reflection from an impermeable wall with an instantaneous twofold increase in pressure, the twofold increase in pressure in the case of wave reflection from a porous wall occurs with a certain delay depending on porosity and permeability of the medium and on fluid viscosity. Therefore, the delay time and the increase in pressure directly after wave incidence onto the wall offer information on material porosity and permeability. The analytical solutions obtained can also be used in testing the results of numerical calculations of pressure waves in saturated porous media.

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REFERENCES

1. Ya. I. Frenkel', "Theory of seismic and seismoelectric phenomena in wet soil," *Izv. Akad. Nauk SSSR, Ser. Geogr. Geofiz.*, **8**, No. 4, 133–150 (1944).
2. M. A. Biot, "Theory of propagation of elastic waves in a fluid-saturated porous solid. 1. Low frequency range," *J. Acoust. Soc. Amer.*, **28**, No. 2, 169–178 (1956).
3. A. Yu. Yumatov and M. G. Markov, "Propagation of an elastic longitudinal wave in a porous crumbling medium," *Geolog. Geofiz.*, No. 3, 98–104 (1987).
4. B. Gurevich and S. Lopatnilov, "Velocity and attenuation of elastic waves in finely layered porous rocks," *Geophys. J. Int.*, **121**, 933–947 (1995).
5. J. A. Hudson, E. Liu, and S. Crampin, "The mechanical properties of materials with interconnected cracks and pores," *Geophys. J. Int.*, **124**, 105–112 (1996).
6. A. A. Gubaidullin and O. Yu. Kuchugurina, "Propagation of weak perturbations in porous crumbling media," *Prikl. Mat. Mekh.*, **63**, No. 5, 816 (1999).
7. V. Sh. Shagapov, I. G. Khusainov, and V. L. Dmitriev, "Propagation of linear waves in gas-saturated porous media with allowance for interphase heat transfer," *J. Appl. Mech. Tech. Phys.*, **45**, No. 4, 552–557 (2004).
8. A. G. Kutushev and S. P. Rodionov, "Numerical study of the effect of the parameters of a bulk layer and an incident shock wave on the pressure at a shielded flat wall," *Combust., Expl., Shock Waves*, **35**, No. 2, 206–213 (1999).
9. A. A. Gubaidullin, D. N. Dudko, and S. F. Urmancheev, "Modeling of the interaction between an air shock wave and a porous screen," *Combust., Expl., Shock Waves*, **36**, No. 4, 496–505 (2000).
10. A. A. Abrashkin, V. S. Averbakh, S. N. Vlasov, et al., "Possible mechanism of the acoustic action on partly saturated porous media," *Akust. Zh.*, **51**, 6–19 (2005).
11. Z. S. Dudin and V. N. Nikolaevskii, "Nonlinear waves in porous media saturated by 'live' oil," *Akust. Zh.*, **51**, 74–80 (2005).
12. V. Yu. Zaitsev, V. É. Gusev, V. E. Nazarov, and B. Castagnede, "Interaction of acoustic waves with cracks: Elastic and inelastic mechanisms of nonlinearity with different time scales," *Akust. Zh.*, **51**, 80–92 (2005).
13. G. A. Maksimov and A. V. Radchenko, "Modeling of oil-production intensification under an acoustic action on the oil bed from the well," *Akust. Zh.*, **51**, 118–132 (2005).
14. M. G. Markov, "Propagation of elastic longitudinal waves in a saturated porous medium with spherical inhomogeneities," *Akust. Zh.*, **51**, 132–140 (2005).
15. V. N. Nikolaevskii and G. S. Stepanova, "Nonlinear seismics and acoustic effect on oil production from an oil bed," *Akust. Zh.*, **51**, 150–160 (2005).
16. G. I. Barenblatt, V. M. Entov, and V. M. Ryzhik, *Motion of Liquids and Gases in Porous Beds* [in Russian], Nedra, Moscow (1984).
17. M. A. Lavrent'ev and V. V. Shabat, *Methods of the Theory of Functions of Complex Variables* [in Russian], Nauka, Moscow (1987).
18. H. Carslaw and J. Jaeger, *Operational Methods in Applied Mathematics*, London (1943).
19. I. I. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, Academic Press, New York (1980).
20. V. I. Krylov and L. T. Shul'gina, *Reference Book on Numerical Integration* [in Russian], Nauka, Moscow (1966).